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Special Theme

Why is Physics Hard? Reflecting on the Pedagogy of Derivations

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Abstract. Derivations are a core component of physics education. In this article, we reflect on some issues and concerns pertaining to the pedagogy of derivations. We then present a teaching narrative and associated interactive learning systems that can potentially address these concerns.

1. Introduction

> *"The day I went into physics class it was death... a man with a high, lisping voice… stood in front of the class… holding a little wooden ball. He put the ball on a steep grooved slide and let it run down to the bottom. Then he started talking about let a equal acceleration and let t equal time and suddenly he was scribbling letters and numbers and equals signs all over the blackboard and my mind went dead. ….. What I couldn't stand was this shrinking everything into letters and numbers… on the blackboard, there were these hideous, cramped, scorpion lettered formulas..."* Sylvia Plath

Physics has the reputation for being difficult, for both students and teachers. This is the case not just in Kerala or India, but across the globe, to varying degrees. The above anecdotal experience shared by Sylvia Plath in her semi-autobiographical work 'The Bell Jar' portrays the feeling of a large section of students attending physics classes. If we informally talk to students in our schools and colleges, we will be able to hear similar accounts. Piles of formulas and technical terms are dumped on them, class after class, as if they have signed up for some occult activity in a strange language. Students are hardly ever explicitly taught the processes through which the real world is shrunk and loaded into mathematical symbols and equations. The connection to the real world and common sense gets obscured by, and lost in, math!

The fact that such an experience holds for a vast majority of students (may be 85 % or even more) makes it a serious educational problem worth pondering about. This article analyzes and discusses some of the possible root sources of this problem. Since the bulk of the time in a typical physics classroom in our country is devoted to teaching derivations, we will focus on derivations.

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2. On the pedagogy of derivations

Derivations are a key component of the instruction and assessment in physics. Though the pedagogy (the way a topic is taught and learned) varies from person to person, we categorize (based on interviews with students and teachers) the approaches of teaching derivations as falling under the two broad categories below.

(a) Mimicking the textbooks: The first approach which we call mimicking the textbooks essentially involves reproducing the same steps and mathematical procedures that are part of the textbook presentation of the derivation. The teacher elaborates or expands upon some of the difficult steps or points, but roughly follows the textbook order. A derivation is perceived here as a series of mathematical steps, procedures and manipulations, resulting in an equation. This equation is then used to solve problems given in the same chapter of the textbook. Not much attention is paid to the connection between the derivation and the real world, or the conceptual dimensions and the physical meaning of the steps and processes involved. This approach leads to students perceiving derivations primarily as a string of mathematical steps connected together, and their learning will be focusing on remembering the key steps and mathematical procedures. Based on this view, their learning mantra becomes lots of practice of the derivations, by doing them multiple times, so that all the steps can be easily recalled during exams.

(b) Representing physical phenomena: In this approach, which we consider as a relatively better one, the teacher makes a connection with the real world physical phenomena that the derivation is referring to. Attention is given to map the mathematical symbols to aspects of the phenomena they are representing. For example, if one is deriving the equation of motion of a pendulum, related examples familiar to students - such as swings, clocks etc - are invoked. Much more sense making happens in this case than the ‘mimicking the textbook’ approach discussed above.

However, even with the better approach (b) above, many fundamental questions which may arise in the mind of students remain unanswered. For example: 1) What is a derivation all about? 2) Why should everything be turned to numbers and letters in physics? Why is idealization done even though it often creates a distorted representation of reality (like treating the sun or earth as a point particle in the derivation of Kepler’s laws) ? 3) What is an equation, and how is it related to the physical phenomena referred to by the derivation ? Questions like these are basically about the nature of the thinking processes underlying derivations. Though the questions seem philosophical in nature, an appreciation of the issues involved is required for understanding the nature of knowledge construction in physics. Such an understanding will give students a better perspective on physics, and help resolve the kind of issues highlighted by the Sylvia Plath quote stated at the beginning of this article. In the next section, we discuss a pedagogical approach and tools we have developed to address these concerns.

3. Derivation as loading of reality into mathematics

We have designed and developed a teaching narrative and associated interactive learning systems,[1] which together present derivations as a process of systematically loading reality into mathematics. The design draws on case studies of the processes by which

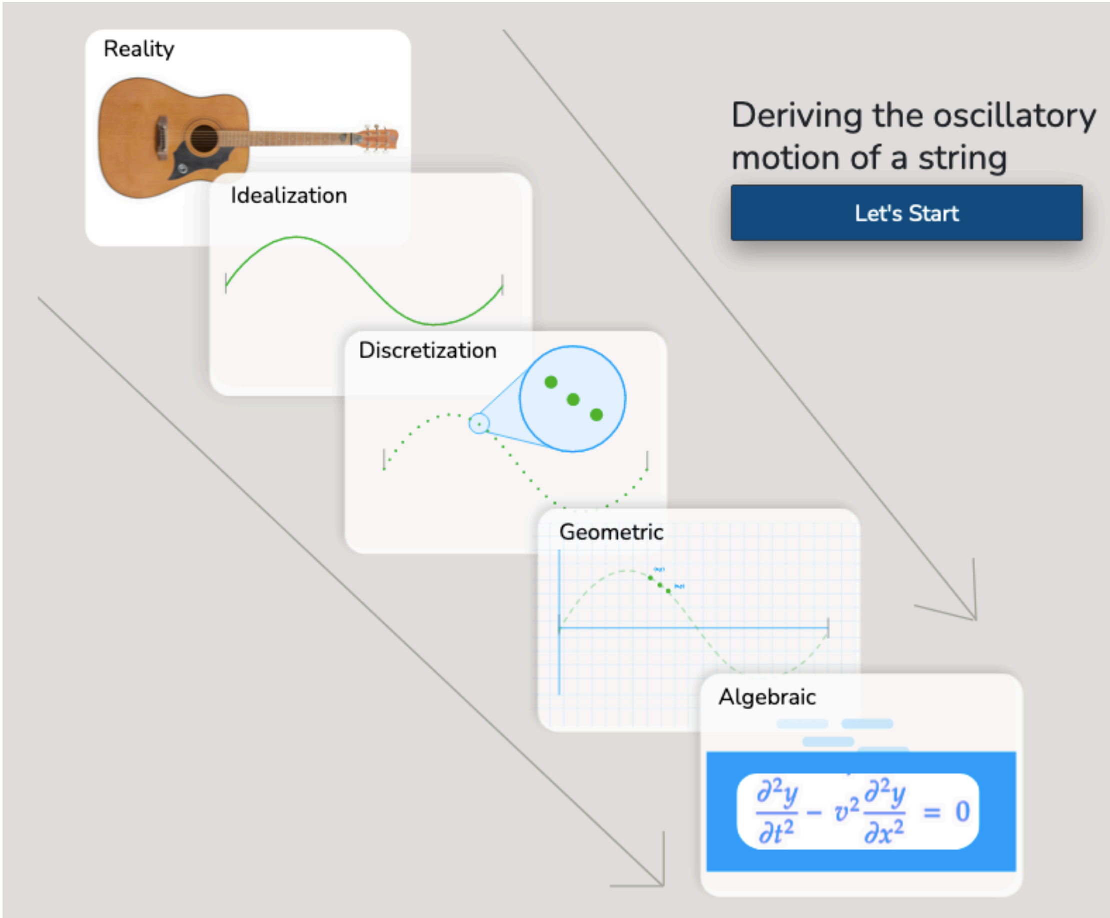

Fig. 1. A snapshot of the interactive derivation system we developed for wave equation. Figure shows the outline of the key steps involved in our approach. The system can be accessed at[1]

physicists like Maxwell and Carnot arrived at mathematical models,[2–4] and insights from our interactions with physics teachers. The interactive system explicitly provides an overall conceptual structure of the processes underlying many derivations in physics. This conceptual structure involves the following steps : Reality → Idealization → Geometric stage → Algebraic stage (See Fig. 1). We illustrate the details of this process using the derivation of the wave equation. The key aspects of our pedagogic narrative, and the design of the learning systems, are as follows:

(1) **Start with an anchoring question**: By this we mean to begin the teaching of a derivation with a question about reality. The derivation is then presented as a mathematical model building activity that aims to answer this question. For example, the following question can anchor the derivation of the wave equation: *A guitar maker want to know about the motion of the guitar string when it is struck. How can you build a formal mathematical system that can help him in this regard?*

(2) **Idealization and decoupling from real world**: The mathematical modeling of the real world system begins by decoupling it from the physical world, and generating a representation that facilitate its analysis.[5] In the case of the guitar string,

the real object is idealized to a 1-D string fixed at both ends. As we can see, many aspects that are deemed irrelevant to the problem at hand are stripped away (color, temperature, material particulars etc) and a mathematical construct (1 D entity) is invoked to represent the system. The ability to make judicious decisions regarding such omissions, and understanding what aspects to focus on, are crucial skills for mathematical modeling. However, usually while teaching derivations, the idealizations is stated as a given, and little reflection or discussion happens on the process by which it is arrived at. Students rarely get the opportunity to be involved in the practice of building an idealization, ie. to make decisions on what to omit and what to focus on, to address the real world problem.

(3) **Discretization**: After idealization we have the 1 D string. Let us see if we can reason out the next steps, rather than recalling them from memory. We know that eventually we want to get an equation that will allow us to predict the motion of the string. For this, we need to know the variation of the position of the different points on the string with time. This implies that if we know the variation of position of an arbitrary point on the string with time, we effectively know about the motion of the whole string. Now the problem is that we have a 1 D string, which is a continuum system, and our goal is to analyse the motion of an arbitrary mass point. To move forward, we have to discretize the string, and consider it as a series of mass points (m) connected together by small springs of lengths dx (in the limit $dx \to 0$). As you can see, this is a crucial modeling move in building the derivation. After descretization, one can focus the analysis on any arbitrary mass point, and eventually reach the goal of arriving at a differential equation characterizing its motion.

(4) **Geometric stage**: After discretization of the string, we need to place the system on a mathematical grid (a coordinate system). This is a crucial step, which allows moving to a mathematical description of the string's motion. This step involves marking the positions and other relevant quantities using mathematical notations. As mentioned before, we are interested in the motion of an arbitrarily chosen mass point on the string. If we want to study motion of a mass, the natural question from a physics standpoint is: What are the forces acting on the mass point ? The mass point is connected by two springs on either side of it. So there are two spring forces acting on the mass point, in addition to gravity. As a simplification, we ignore gravity as it can be considered negligible compared to the spring forces.

(5) **Algebraic stage**: We have learned the expression for force due to extension in spring ($f = -kx$) in class 11. Feeding in the details from the geometrical representation into $f = -kx$, and summing up the two forces results in the expression for the total force (F) on the mass point, which is given by,

$$F = k_1(Y_{x+dx} - 2Y_x + Y_{x-dx}) \tag{1}$$

The details of the steps involved are available in our system.[1] From our experience we know that, at some point in a derivation, fundamental equations - like $F = ma$, Maxwell's equations or the Schrodinger equation - make their entry. Often this step involves conjoining the details of the particular system under analysis with the basic equation, to generate a differential equation. In the case of our guitar string system, the fundamental equation would be $F = ma$, as it is a mechanical system. However to conjoin the string system with $F = ma$ we first need to turn the difference equation 1 into a differential equation. This can be done with the help of a theorem from calculus (Refer our system for details[1]).

$$F = k_1(Y_{x+dx} - 2Y_x + Y_{x-dx}) = k_1 \frac{d^2y}{dx^2}(dx)^2 \tag{2}$$

Conjoining Eq. 2 with $F = ma$ and rearranging the terms we get the wave equation, given by

$$\frac{\partial^2 y}{\partial t^2} - v^2 \frac{\partial^2 y}{\partial x^2} = 0 \tag{3}$$

Here $v = \sqrt{\frac{k_1 dx^2}{m}}$ (refer[1] for details). Eq. 3, on solving, will enact ('act out') the behavior of the string we started with. In other words, we have loaded the reality (motion of guitar string) into mathematics (wave equation), through a series of steps and modeling moves.

In hindsight we can see that the idealization and discretization we carried out in the initial stages were in fact guided or dictated by Newton's second law. Thus the derivation process is not strictly linear. The linear approach we presented is used for pedagogical purposes. Note that the conceptual schema we presented above (Reality → Idealization → Geometric stage → Algebraic stage) is not limited to this particular derivation, but is applicable to many other derivations. An interesting task you can do to improve your teaching is to see how this schema could be used to restructure other derivations students find difficult. If you develop such a structure for a new and difficult topic, please get in touch with us, and we can explore turning this structure into an interactive system similar to the wave equation one. To get a richer feel for the extended and dynamic nature of the thinking processes involved in building mathematical models, we recommend reading cognitive science case studies of mathematical modeling, particularly ones describing how Maxwell went about modeling the EM field, and Carnot, his ideal heat engine.[2,3]

4. Concluding remarks

To conclude, physics as a discipline is quintessentially concerned with building enactive mathematical models of the physical world, particularly to capture dynamics, in a way that the mathematical system can 'act out' the dynamic behavior we are interested in. This 'acting out' nature of the equation allows us to make predictions about the dynamics of the real world system we start with. The 'shrinking of everything into letters and numbers' mentioned in the beginning quote by Sylvia Plath is thus done to make predictions about dynamics, and such shrinking is unavoidable in the process of knowledge construction in physics. To mitigate the discomfort and difficulty experienced by many students in this process of condensing reality into symbols, what we can do is to explicitly instruct them on why and how this shrinking everything into letters and numbers is done. Once they are aware of the rationale, necessity, and the systematic sequence of modeling steps underlying the whole process, they may be less likely to consider physics as a jumble of equations and terms, and thus get beyond the mind numbing experience! Making explicit the modeling moves underlying derivations, along the approach we described, have the following additional advantages as well:

(1) Students need not remember and recall each and every step of all derivations. As we have described, there is a structure and logic for the whole process, and knowing this structure allows the student to reason out the rationale for most of the steps.

(2) The conceptual structure we have elucidated for wave equation (Reality $\rightarrow$ Idealization $\rightarrow$ Geometric stage $\rightarrow$ Algebraic stage) is a general one. On reflection, one can easily see that the same conceptual structure is applicable for many other derivations. Such a realization will help in our knowledge organization. If we are learning say 100 derivations, they don't have to be considered as 100 unrelated activities. Rather they can be conceived as 100 similar ways of loading reality into mathematics!

(3) Perceiving derivations as loading of reality into mathematics will help students tackle novel and unfamiliar problems better. If they perceive their physics experience as loading 100 instances of reality into mathematics, they are less likely to be at a total loss when confronted with a 101th novel case !

(4) Treating derivations as a mathematical modeling activity can facilitate a smooth transition to novel approaches to modeling, such as building computational simulations and also building models to solve complex interdisciplinary problems, such as the virus transmission model. Interested readers may refer to[6] for further details on this (we have been conducting a series of 2 day workshops for higher secondary and UG teachers in Kerala on this approach).

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